

Elliptically polarized electromagnetic waves in a magnetized quantum electron-positron plasma with effects of exchange-correlation

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The dispersion properties of elliptically polarized electromagnetic (EM) waves in a magnetized electron-positron-pair (EP-pair) plasma are studied with the effects of particle dispersion associated with the Bohm potential, the Fermi degenerate pressure, and the exchange-correlation force. Two possible modes of the extraordinary or X wave, modified by these quantum effects, are identified and their propagation characteristics are investigated numerically. It is shown that the upper-hybrid frequency, and the cutoff and resonance frequencies are no longer constants but are dispersive due to these quantum effects. It is found that the particle dispersion and the exchange-correlation force can have different dominating roles on each other depending on whether the X waves are of short or long wavelengths (in comparison with the Fermi Debye length). The present investigation should be useful for understanding the collective behaviors of EP plasma oscillations and the propagation of extraordinary waves in magnetized dense EP-pair plasmas.

I. INTRODUCTION

Study of electron-positron (EP) plasmas provides important insights about the early universe^{1,2} and astrophysical objects (such as supernova remnants, pulsars, active galactic nuclei etc.)³⁻⁵. Such plasmas may be created by different physical mechanisms, for example, collisions between the accelerated particles, streaming of charged particles along the curved magnetic fields and the interaction of high-energy lasers with plasmas. Over the past few years, a number of experiments have been proposed to create EP plasmas (see, e.g., Refs. 6–8). However, because of the fast annihilation and the formation of positronium atoms, the identification of collective modes in these EP plasmas may be practically impossible. To resolve this issue, some attempts have been made to produce high-density ($\sim 10^{16} \text{ cm}^{-3}$) neutral EP plasmas (ion-free plasmas with unique characteristics) in the laboratory⁹.

On the other hand, it has been shown that the propagation of electromagnetic (EM) radiation in EP plasmas, is responsible for the high effective temperatures of pulsar radio emissions¹⁰⁻¹². The knowledge on the dynamics of such EM waves in EP plasmas is essential for understanding the radiation properties of astrophysical objects, even the media exposed to the field of super-strong laser radiation¹³. These may be the reason behind the attraction of attention on the investigation of collective phenomena in EP plasmas. Several authors have investigated the propagation characteristics of EM waves in EP plasmas (see, e.g., Refs. 14–18). It is well known that the quantum effects can play a vital role in plasmas

when the de Broglie wavelength becomes comparable to the average inter-particle distance¹⁹. In such systems, the quantum effects significantly change the collective behaviors of plasma species²⁰⁻²³. For instance, the Bohm potential leads to appearance of higher-order corrections in the dispersion relation, meanwhile the spin magnetization contributes to the linear dispersion of electromagnetic modes^{24,25} etc. Several theoretical models have been proposed to study the collective behaviors and associated nonlinear structures in quantum plasmas²⁴. Furthermore, the quantum magnetohydrodynamic (QMHD) model has also been extensively used to study the influence of quantum effects on the wave propagation in quantum plasmas²⁵⁻²⁹. Shukla²² investigated the combined effects of the quantum Bohm potential and the electron spin-1/2 effects on the extraordinary (X) EM (X-EM) waves in a warm dense magnetoplasma. He showed that the quantum effects significantly modify the dispersion properties of such waves. The influence of electron magnetization spin current on the X-EM waves in a magnetized quantum plasma has also been examined by Li *et. al.*³⁰.

It has been shown that the effects of exchange and correlation forces on the plasma collective oscillations can be significant and can even play dominant roles over the particle dispersion³¹. To complement and give new insights into the previously published works³⁰, we propose here to address the propagation properties of X-EM waves in a magnetized quantum EP plasma including the combined influences of the exchange-correlation potential and the Bohm potential. In order to describe the present plasma system, we employ the QMHD model, and obtain the modified dispersion relation. It is shown that the quantum effects modify the dispersion properties of X-EM waves together with the cutoff and resonance frequencies. The manuscript is organized as follows: The basic equations governing dynamics of EP-pair plasmas are given

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in Sec. II. The dispersion relation of the X-EM waves is derived in Sec. III. Our theoretical results and a brief discussion are provided in Sec. IV. Finally, we conclude our results in Sec. V.

II. THEORETICAL MODEL

We consider a magnetized quantum plasma consisting of degenerate electrons as well as positron fluids in presence of an uniform external magnetic field acting along the z -axis, i.e., $\mathbf{B}_0 = B_0 \hat{z}$. The QMHD model (which can be obtained from the self-consistent Hartree equations¹⁹ or from the phase-space Wigner-Poisson equations³²) describing the dynamical behaviors of electrons and positrons is given by

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0, \quad (1)$$

$$m \frac{\partial \mathbf{v}_j}{\partial t} = q_j (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}) - \frac{\nabla P_j}{n_j} - \nabla V_{qj} - \nabla V_{xj}, \quad (2)$$

where n_j , \mathbf{v}_j and q_j , respectively, denote the number density, velocity and charge of j -species particle, and $m_e = m_p = m$ is the mass of electrons ($j = e$) and positrons ($j = p$). Also, \mathbf{E} (\mathbf{B}) is the electric (magnetic) field and ϵ_0 is the permittivity of free space. The equations (1)-(2) are closed by the Maxwell's equations, to be presented shortly. Furthermore, P_j is the weakly relativistic pressure for degenerate electrons and positrons given by³³

$$P_j = \frac{1}{5} \frac{m V_F^2}{n_0^{2/3}} n_j^{5/3}, \quad (3)$$

where $V_F \equiv \sqrt{2k_B T_F / m} = (\hbar/m)(3\pi^2 n_0)^{1/3}$ is the Fermi velocity with k_B denoting the Boltzmann constant, T_F the Fermi temperature and n_0 the equilibrium number density of electrons and positrons. The particle dispersion associated with the density correlation due to quantum fluctuation (tunneling) is included in Eq. (2) via the Bohm potential V_{qj} given by¹⁹

$$V_{qj} = -\frac{\hbar^2}{2m} \frac{1}{\sqrt{n_j}} \nabla^2 \sqrt{n_j}. \quad (4)$$

Furthermore, in dense plasmas the electron/positron exchange/correlation effects play a non-negligible role³¹. The general framework that produces such exchange-correlation potentials is the density functional theory (DFT)³⁴. This potential is a complicated function of the particle number density given by³⁵⁻³⁸

$$V_{xj} = -0.985 \left(n_j^{1/3} e^2 / \epsilon \right) \left[1 + \left(0.034 / n_j^{1/3} a_B \right) \times \ln \left(1 + 18.376 n_j^{1/3} a_B \right) \right], \quad (5)$$

where ϵ is the effective dielectric constant of the medium and $a_B = \epsilon \hbar^2 / m e^2$ is the Bohr radius.

Next, linearizing the set of equations (1) and (2) by splitting up the physical quantities into their equilibrium (with a zero value or a quantity with suffix 0) and perturbation parts (with suffix 1), i.e., $n \sim n_0 + n_1$, $\mathbf{v} \sim \mathbf{0} + \mathbf{v}_1$, $\mathbf{E} \sim \mathbf{0} + \mathbf{E}_1$ and $\mathbf{B} \sim \mathbf{B}_0 + \mathbf{B}_1$, we obtain (after omitting the suffix 1)

$$\frac{\partial n_j}{\partial t} + n_0 \nabla \cdot \mathbf{v}_j = 0, \quad (6)$$

$$\frac{\partial \mathbf{v}_j}{\partial t} = \frac{q_j}{m} (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}_0) - \frac{1}{3} \frac{\alpha}{n_0} \nabla n_j + \frac{\beta}{n_0} \nabla (\nabla^2 n_j), \quad (7)$$

where $\beta = \hbar^2 / 4m^2$ is the coefficient of the term associated with the Bohm potential and

$$\alpha = V_F^2 \left[3 - (3\pi^2)^{2/3} H^2 \left(0.985 + \frac{0.616}{1 + 1.9/H^2} \right) \right], \quad (8)$$

in which the first term in the square brackets appears due to the weakly relativistic degenerate pressure of electrons and positrons, and the second term $\propto H$ is due to the exchange and correlation effects. Here, $H = \hbar \omega_p / m V_F^2 \equiv \hbar \omega_p / 2k_B T_F$ is the dimensionless quantum parameter measuring the ratio of the electron/positron plasmon energy to the Fermi energy densities. Note that for high-density plasmas, $H \lesssim 1$, and so the expression (8) can be further simplified as

$$\alpha \approx V_F^2 \left[3 - (3\pi^2)^{2/3} H^2 \right]. \quad (9)$$

The perturbed electromagnetic fields are governed by the Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (10)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}, \quad (11)$$

where $\mathbf{J} = en_0(\mathbf{v}_p - \mathbf{v}_e)$ is the current density. In what follows, we consider an elliptically polarized electromagnetic wave propagating along the x -axis. We suppose that the polarized electric field lies in xy -plane, i.e., $\mathbf{E} = E_x \hat{x} + E_y \hat{y}$, and as before the external magnetic field is along the z -axis, i.e., $\mathbf{B}_0 = B_0 \hat{z}$. Assuming the variation of the perturbed quantities to be of the form $\sim \exp(ikx - i\omega t)$ with k (ω) denoting the wave number (frequency) of perturbations, we obtain from Eqs. (6) and (7) the following equations

$$\omega n_j = n_0 \mathbf{k} \cdot \mathbf{v}_j, \quad (12)$$

$$\omega \mathbf{v}_j = i \frac{q_j}{m} (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}_0) + \frac{\zeta k}{n_0} n_j \hat{x}, \quad (13)$$

where $\zeta = V_F^2 (1 - 3.2H^2 + \frac{1}{4}H^2 k^2 \lambda_F^2)$ with $\lambda_F = V_F/\omega_p$ denoting the Fermi Debye length. Using Eq. (12) and separating the x - and y -components of Eq. (13) we obtain

$$\left(1 - \zeta \frac{k^2}{\omega^2}\right) v_{jx} \pm \frac{i\omega_c}{\omega} v_{jy} = \mp \frac{ie}{m\omega} E_x, \quad (14)$$

$$\frac{i\omega_c}{\omega} v_{jx} \mp v_{jy} = \frac{ie}{m\omega} E_y, \quad (15)$$

where the upper and lower signs in \pm or \mp refer to the signs of charges of electrons and positrons respectively. Next, from Eqs. (10) and (11) we obtain

$$i\omega E_x = \frac{m}{e} \omega_p^2 (v_{px} - v_{ex}), \quad (16)$$

$$i(\omega^2 - c^2 k^2) E_y = \frac{m}{e} \omega_p^2 (v_{py} - v_{ey}). \quad (17)$$

From Eqs. (14) and (15) solving for v_{jx} and v_{jy} we obtain

$$v_{jx} = \frac{(e/m)\omega}{\omega^2 - \omega_c^2 - \zeta k^2} \left(\pm i E_x - \frac{\omega_c}{\omega} E_y \right), \quad (18)$$

$$v_{jy} = \frac{(e/m)}{\omega^2 - \omega_c^2 - \zeta k^2} \left(\omega_c E_x \pm i \frac{\omega^2 - \zeta k^2}{\omega} E_y \right). \quad (19)$$

III. DISPERSION RELATION

Substituting the expressions of v_{jx} and v_{jy} from Eqs. (18) and (19) into Eqs. (16) and (17), we obtain for nonzero values of E_x and E_y the following dispersion relations for the quantum modified X-wave and the upper-hybrid wave

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{2\omega_p^2}{\omega^2} \left(\frac{\omega^2 - \zeta k^2}{\omega^2 - \omega_c^2 - \zeta k^2} \right), \quad (20)$$

$$\omega^2 = \omega_{uh}^2 + \zeta k^2, \quad (21)$$

where $\omega_{uh} = \sqrt{2\omega_p^2 + \omega_c^2}$ is the upper-hybrid oscillation frequency in classical plasmas. By disregarding the quantum effects $\propto \zeta$, one can obtain from Eq. (20) the dispersion relation for X-EM waves in electron-positron plasmas as

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{2\omega_p^2}{\omega^2} \left(\frac{\omega^2}{\omega^2 - \omega_c^2} \right), \quad (22)$$

and from Eq. (21) the usual hybrid frequency $\omega = \omega_{uh}$. Also, by neglecting the quantum effects and the positron dynamics, and assuming positive ions to form the background plasma, one can obtain the known classical results for the X-wave as³⁹

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \left(\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_{uh}^2} \right). \quad (23)$$

Now, comparing Eq. (20) with Eq. (23), we find that the dispersion of the X-EM wave in EP plasmas is greatly modified by the quantum effects $\propto \zeta$. Here, in Eq. (20), the factor 2 appears due to pair plasmas with equal mass and unperturbed number density. Also, after substitutions of the solutions (18) and (19) into Eqs. (16) and (17), we note that (because of pair plasmas) the corresponding terms $\propto E_y$ and E_x in these equations cancel each other. As a result the term ω_p^2 disappears from both the numerator and denominator of the term in the parentheses in Eq. (20). Furthermore, the dispersion equation (20), upon dropping the positron dynamics (hence the corresponding factor and the terms as above) and the contribution from the exchange correlation force, agrees with that in Ref. 30 except the spin magnetization current which has not been considered in the present model.

In what follows, we recast Eq. (20) into its dimensionless form as

$$\frac{c_0^2 K^2}{\Omega^2} = 1 - \frac{2}{\Omega^2} \frac{\Omega^2 - \zeta_0 K^2}{\Omega^2 - \Omega_c^2 - \zeta_0 K^2}, \quad (24)$$

where $c_0 = c/V_F$, $\zeta_0 = \zeta/V_F^2$, $K = k\lambda_F$, $\Omega = \omega/\omega_p$ and $\Omega_c = \omega_c/\omega_p$. Note that in Eq. (24), the influence of the quantum effects is mediated through the terms proportional to ζ_0 both in the numerator and denominator. Here, we recall that the first term in ζ_0 appears due to the degeneracy pressure of electrons and positrons, the second term is due to the exchange-correlation force and the third term is for the particle dispersion associated with the Bohm potential. Furthermore, both the exchange correlation and the particle dispersive effects scale as $\sim H^2$. Thus, it follows that in the long-wavelength EP plasma oscillations with $K \ll 1$, the quantum dispersive effects can be negligible compared to the contribution from the exchange-correlation force³¹. However, for short-wavelength oscillations ($K \gg 1$), the effects can be reverse, i.e., the particle dispersion may be comparable to or even dominate over the exchange-correlation force. However, for long-wavelength EP plasma oscillations in dense environments where $H < 1$, the degeneracy pressure may play a dominating role over the exchange-correlation force as well as the particle dispersion. In the next sections we will analyze these effects numerically on the wave dispersion as well on the phase and group velocities of the X wave.

Equation (24), while expressed in Ω , gives two modes of the X-wave

$$\Omega_{XL, XU}^2 = \frac{1}{2} \left[\Omega_X \mp \sqrt{\Omega_X^2 - 4(2\zeta_0 K^2 + c_0^2 K^2 \Omega_{res}^2)} \right], \quad (25)$$

where $\Omega_X = \Omega_{res}^2 + c_0^2 K^2 + 2$ and $\Omega_{res}^2 = \Omega_c^2 + \zeta_0 K^2$, to be shown as the square of the resonance frequency shortly.

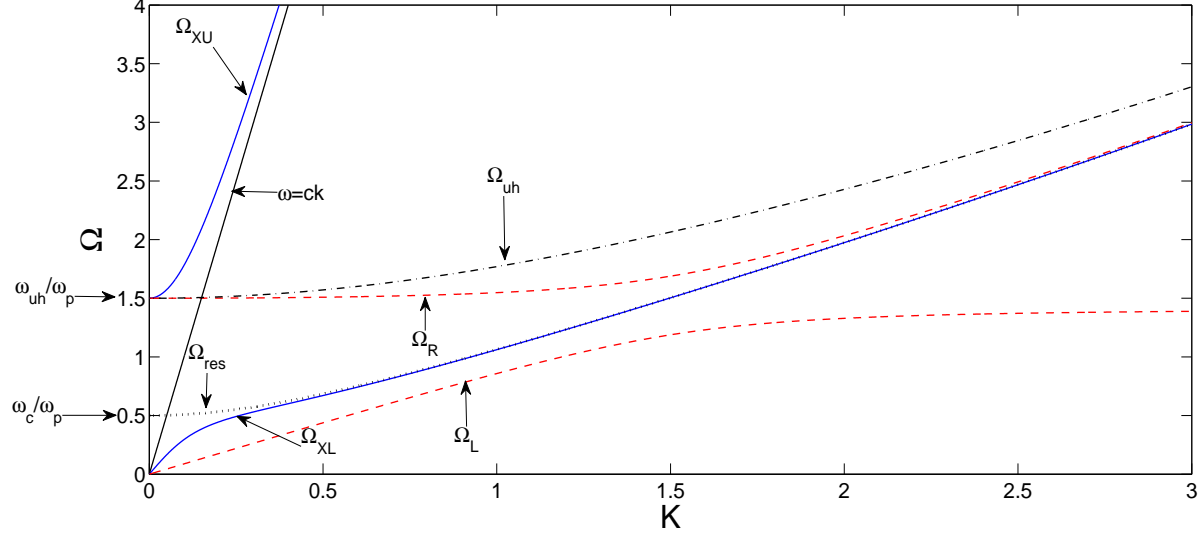


FIG. 1. The dispersion of the X-wave together with the cutoff and resonance frequencies are shown. While the solid lines (except the straight line which represents $\omega = ck$) represent the two branches of the X-wave (Ω_{XL} and Ω_{XU}), the dashed lines correspond to the left- (Ω_L) and right-hand (Ω_R) cutoff frequencies. The dotted and dash-dotted lines, respectively, correspond to the resonance frequency (Ω_{res}) and the upper-hybrid wave frequency (Ω_{uh}). The parameter values are $\Omega_c = 0.5$ and $H = 0.2$.

A. Phase velocity and group velocity of X-wave

The expressions for the phase velocity (ω/k) and the group velocity ($d\omega/dk$) can be obtained from the dispersion relation (24) by a straightforward algebra as

$$V_p \equiv \frac{\omega}{kV_F} = c_0 \left(1 - \frac{2}{\Omega^2} \frac{\Omega^2 - \zeta_0 K^2}{\Omega^2 - \Omega_{res}^2} \right)^{-1/2}, \quad (26)$$

$$V_g \equiv \frac{1}{V_F} \frac{d\omega}{dk} = \frac{\Omega_1 (\zeta_0 + H^2 K^2/4) - \Omega_2 c_0^2}{V_p (\Omega_1 - \Omega_2)}, \quad (27)$$

where $\Omega_1 = 2 - \Omega^2 + c_0^2 K^2$ and $\Omega_2 = \Omega^2 - \Omega_{res}^2$. Note that corresponding to two different X-wave modes given by Eq. (25), and using Eqs. (26) and (27) one can obtain different expressions for the phase velocity and group velocity. In Sec. IV we will see that while the group velocity of the branch Ω_{XL} increases with $K > 1$, the upper branch Ω_{XU} approaches a constant value for $K > 1$.

B. Cutoff and resonance of X-wave

The dispersion relation of the X-wave is somewhat complicated. However, to analyze its properties it is useful to obtain the cutoff and resonance frequencies. Such cutoffs occur when the index of refraction ck/ω goes to zero or when the wavelength becomes infinite. On the other hand, the resonance of the X-wave occurs when the index of refraction becomes infinite or the wavelength becomes zero. The X-wave will then be reflected at the

cutoff frequency and absorbed at the resonance. Thus, from Eq. (24) the cutoff frequencies can be obtained as

$$\Omega_{R,L}^2 = \frac{1}{2} \left[\Omega_{uh}^2 \pm \sqrt{\Omega_{uh}^4 - 8\zeta_0 K^2} \right], \quad (28)$$

where $\Omega_{uh}^2 = \Omega_c^2 + \zeta_0 K^2 + 2$ is the quantum modified hybrid frequency, and the suffices R and L , corresponding to the plus and minus sign, respectively, stand for the right- and left-hand cutoff frequencies of the X-wave modified by the quantum effects. Note that in absence of the quantum effects, while the left-hand cutoff occurs at a zero value, the right-hand cutoff occurs at the upper-hybrid frequency. The resonance frequency is obtained from Eq. (24) as

$$\Omega_{res}^2 = \Omega_c^2 + K^2 \left(1 - 3.2H^2 + \frac{1}{4}H^2 K^2 \right). \quad (29)$$

Thus, the resonance frequency of the X-EM wave is also modified by the quantum effects, in absence of which the resonance occurs at the cyclotron frequency instead of the upper-hybrid frequency as in classical X-Em waves³⁹.

IV. RESULTS AND DISCUSSION

We numerically investigate the dispersion properties of the X-wave [Eq. (25)] through the behaviors of the cutoff and resonance frequencies given by Eqs. (28) and (29). The latter are important in part as they define the pass and stop bands where the X-EM waves can propagate in EP plasmas. These are illustrated in Fig. 1. It clearly

shows that the X waves (solid lines except that represents $\omega = ck$) can propagate in the pass band regions of $\Omega_L < \Omega < \Omega_{res}$ and $\Omega > \Omega_{uh}$, however, a stop band exists in the range $\Omega_{res} < \Omega < \Omega_R$. The cutoffs (dashed lines) are at the frequencies where the dispersion relation vanishes into the refractive index $ck/\omega = 0$, and the resonances (dotted line) are where $ck/\omega \rightarrow \infty$. We also note that while for the upper pass band $V_p > c_0$, the lower branch of the X-wave propagates with a phase velocity (V_p) smaller than the speed of light in vacuum (c_0). Furthermore, because of the quantum effects, namely the degeneracy pressure, the exchange-correlation and the particle dispersion, the upper-hybrid frequency, and the cutoff and resonance frequencies are no longer constants but are dispersive and increase with the wave number K . Furthermore, it is also seen from Fig. 1 that in contrast to classical EM wave, the group velocities of the X waves do not tend to vanish in the vicinity of the upper-hybrid frequency.

The properties of the group velocities of the two branches of the X wave are shown in Fig. 2 for different values of the quantum parameter H and the cyclotron frequency Ω_c . It is seen that while the parameter H does not have any influence on the upper branch Ω_{XU} [see the right panel (b)], it can reduce the group velocity of the lower branch Ω_{XL} in the range $0 < K < 2$ and increase in the other regime [see the left panel (a)]. Also, V_g of the branch Ω_{XU} approaches a constant value, whereas that of Ω_{XL} keeps increasing with $K > 1$. Furthermore, the effect of the cyclotron frequency on V_g of Ω_{XL} is to increase its value in the regime of $0 < K \lesssim 1$ and decrease in $K \gtrsim 1$. However, increasing values of the same (Ω_c) decreases the values of V_g of Ω_{XU} .

Figure 3 clearly shows how the quantum parameter H and the the cyclotron frequency Ω_c greatly influence the characteristics of the lower/upper branches as well as the cutoff frequencies of the X-EM wave. It is seen that the values of Ω_{XU} remain unchanged with H , whereas the other frequencies get significantly modified by this parameter. Note that in most the cases, except for Ω_L , the effect of Ω_c is to increase the frequencies. It is also seen that Ω_L approaches a constant value at large K and the effects of Ω_c on the frequencies become significant in the regime of $K \rightarrow 0$. It is also found that the behaviors of the lower branch Ω_{XL} of the X-wave are almost similar to those of the right-hand cutoff frequency Ω_R [see panels (a) and (d)].

To investigate the influence of different quantum forces separately and as illustrations we have plotted Ω_L and Ω_{XL} (Here, we recall that Ω_{XL} has similar behaviors as of Ω_R and the quantum parameter H does not have any significant effect on Ω_{XU}) with K as in Fig. 4. It is shown that for the left-hand cutoffs the correlation effect can be negligible for higher values of K or for short-wavelength oscillations ($K \gg 1$), however, its effect is no longer negligible for moderate values of K or as $K \rightarrow 0$ (see the solid and dashed lines). For the right-hand cutoff or the lower branch of the X-wave, the effect of

the correlation force is significant in the range $K > 1$. In both the panels (a) and (b) it is also seen that the particle dispersion dominates over the correlation force and the degeneracy pressure in the range $K > 1$.

V. CONCLUSION

We have investigated the dispersion properties of the X-EM waves in a magnetized degenerate EP-pair plasma with the effects of weakly relativistic degeneracy pressure, the quantum force associated with the Bohm potential and that due to exchange and correlations of electrons and positrons. It is shown that the latter two effects, which scale as H (the ratio of plasmon energy to the Fermi energy densities) along with the degeneracy pressure significantly modify the refractive index of the X-wave. It is also seen that while the particle dispersion becomes important for short-wavelength oscillations, the exchange correlation can no longer be negligible (or can even dominate over the other quantum effects) in the long-wavelength perturbations. The upper-hybrid frequency together with the cutoff and resonance frequencies are also shown to be dispersive as modified by the quantum effects. Furthermore, in contrast to the classical X-EM waves, the group velocity is shown to be non-vanishing in the vicinity of the upper-hybrid frequency. To conclude, the results should be useful for understanding the dispersion properties of X-EM waves that can propagate in magnetized dense EP-pair plasmas such as those in magnetized white dwarf stars, neutron stars etc.

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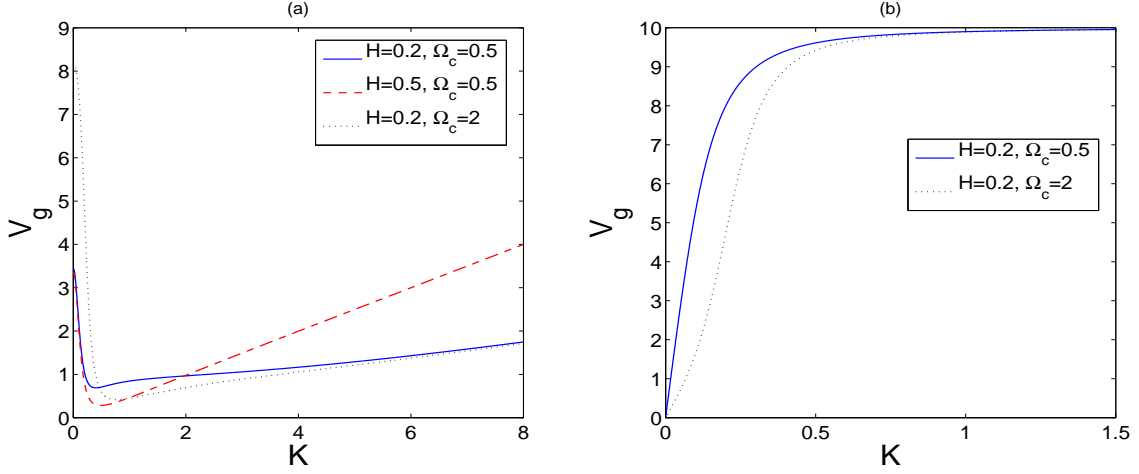


FIG. 2. Group velocities of the X wave [Eq. (27)] are plotted against the wave number K for different values of the quantum parameter H and the cyclotron frequency Ω_c as in the legends. The left and right panels (a) and (b) are corresponding to the lower and upper branches Ω_{XL} and Ω_{XU} of the X-wave respectively.

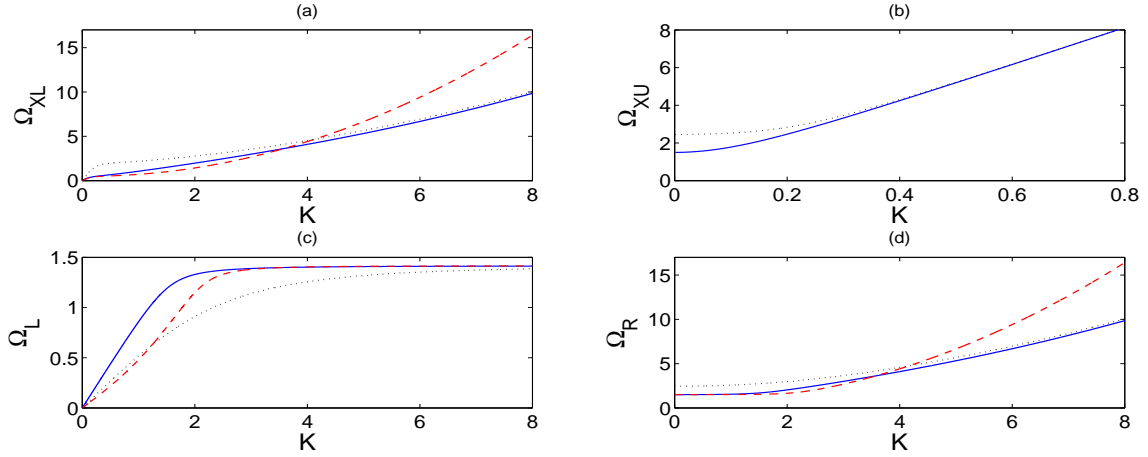


FIG. 3. Two branches (Ω_{XL} , Ω_{XU}) of the X-wave [see the upper panels (a) and (b)] and two cutoff frequencies (Ω_L , Ω_R) [see the lower panels (c) and (d)] are plotted against the wave number K for different values of the quantum parameter H and the cyclotron frequency Ω_c as in Fig. 2, i.e., $H = 0.2$, $\Omega_c = 0.5$ (solid line); $H = \Omega_c = 0.5$ (dashed line) and $H = 0.2$, $\Omega_c = 2$ (dotted line). It is seen that the quantum parameter H does not have any significant effect on the upper branch (Ω_{XU}) of the X-wave.

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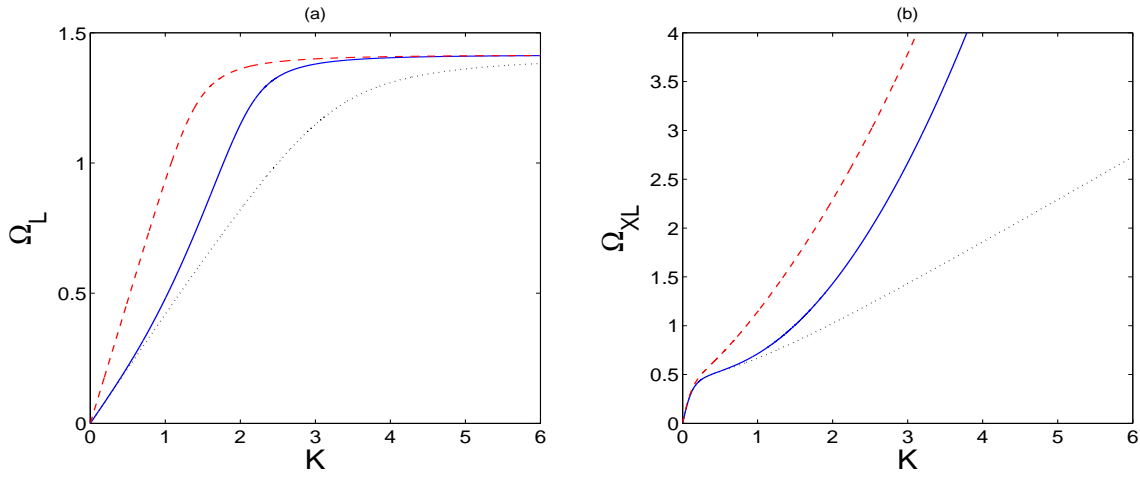


FIG. 4. The left-hand cutoff frequency [Ω_L in Eq. (28)] and the lower branch of the X-wave [Ω_{XL} in Eq. (25)] are plotted to show the effects of different quantum parameters. The solid line is such when all the three effects, namely the degenerate pressure, the exchange-correlation and the quantum dispersion are present ($H = \Omega_c = 0.5$). The dashed line corresponds to the effects of the degenerate pressure and the quantum dispersion, and in absence of exchange-correlation effect. The dotted line corresponds to the effects of the degenerate pressure and the exchange-correlation force, and in absence of the quantum dispersion. Note that Ω_R has similar behaviors as of Ω_{XL} with the quantum parameter H , and Ω_{XU} remains almost unchanged with H .

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